

التعارين ع-1-د

اجب بصواب او خطأ مع تعليل الإجابة في كل حالة

$$\left(\frac{5}{\sqrt{3}}\right)^{-2} = \left(\frac{\sqrt{3}}{5}\right)^2 = \frac{3}{25} \text{ صواب لأن } \left(\frac{5}{\sqrt{3}}\right)^{-2} = \frac{3}{25} \quad (1)$$

$$-3(\sqrt{5})^3 = -3 \times 5\sqrt{5} = -15\sqrt{5} \text{ صواب لأن } -3\sqrt{5^3} = -15\sqrt{5} \quad (2)$$

$$\left(-\frac{\pi}{3}\right)^{-11} = \left(-\frac{3}{\pi}\right)^{11} \text{ خطأ لأن } \left(-\frac{\pi}{3}\right)^{-11} = \left(\frac{3}{\pi}\right)^{11} \quad (3)$$

$$\sqrt{7^6} = 7^3 \text{ و } \sqrt{7^2} + \sqrt{7^4} = 7 + 7^2 \text{ خطأ لأن } \sqrt{7^2} + \sqrt{7^4} = \sqrt{7^6} \text{ و } 7^3 \neq 7 + 7^2 \quad (4)$$

$$\frac{\sqrt{3^{45}}}{\sqrt{3^5}} = \sqrt{\frac{3^{45}}{3^5}} = \sqrt{3^{45-5}} = \sqrt{3^{40}} \text{ خطأ لأن } \frac{\sqrt{3^{45}}}{\sqrt{3^5}} = \sqrt{3^9} \quad (5)$$

$$\sqrt{(10^{-4})^2} = (\sqrt{10^{-4}})^2 = 10^{-4} \text{ و } 10^{-8} = (10^{-4})^2 \text{ صواب لأن } \sqrt{10^{-8}} = 10^{-4} \quad (6)$$

التعارين ع-2-د

اختر الإجابة الصحيحة من بين الإجابات التالية علما وأنه واحدة فقط هي صحيحة

$$(1) \text{ العدد } (-\sqrt{2})^{-3} \times \left(-\frac{1}{\sqrt{2}}\right)^3 \text{ يساوي : (أ) } \frac{1}{8} \text{ (ب) } 1 \text{ (ج) } -1$$

$$(2) \text{ العدد } 7(\sqrt{7})^{-2} \text{ يساوي : (أ) } \frac{1}{(7\sqrt{7})^2} \text{ (ب) } \frac{1}{49} \text{ (ج) } 1$$

$$(3) \text{ يساوي } \left(-\frac{6}{7}\right)^{-15} \text{ (أ) } \left(\frac{6}{7}\right)^{15} \text{ (ب) } \left(\frac{7}{6}\right)^{15} \text{ (ج) } \left(-\frac{7}{6}\right)^{15}$$

$$(4) \text{ العدد } 0,003478 \text{ يساوي}$$

$$\text{(أ) } 3,487 \times 10^{-6} \text{ (ب) } 3,487 \times 10^{-3} \text{ (ج) } 3,487 \times 10^3$$

$$(5) \left[(\sqrt{2})^{-2}\right]^4 \times 125 - \left[(\sqrt{2})^{-2}\right]^4 \times 100 \text{ يساوي}$$

$$\text{(أ) } 25 \text{ (ب) } 16 \times 25 \text{ (ج) } \left(\frac{\sqrt{5}}{2}\right)^4$$

$$(6) (\sqrt{3})^{-2} + (-\sqrt{3})^2 \text{ يساوي (أ) } \frac{10}{3} \text{ (ب) } 1 \text{ (ج) } 0$$

التعارين ع-3-د

أجب

$$(-3\sqrt{5})^3 = -27 \times 5\sqrt{5} = -135\sqrt{5} >$$

$$\left(\frac{2}{\sqrt{7}}\right)^{-4} = \left(\frac{\sqrt{7}}{2}\right)^4 = \frac{49}{16} >$$

$$3\sqrt{5^{-4}} = 3(\sqrt{5})^{-4} = 3 \times \frac{1}{(\sqrt{5})^4} = \frac{3}{25} >$$

$$\frac{(5870)^0}{\frac{(3\sqrt{3})^{-4}}{10^{-3}}} = \frac{1}{\frac{10^3}{(3\sqrt{3})^4}} = \frac{(3\sqrt{3})^4}{10^3} = (3\sqrt{3})^4 \times 10^{-3} = 729 \times 10^{-3} = 0.729 \quad \triangleright$$

$$\left(\frac{3}{2}\right)^{-2} + \left(\frac{6}{5}\right)^{-1} = \left(\frac{2}{3}\right)^2 + \left(\frac{5}{6}\right)^1 = \frac{4}{9} + \frac{5}{6} = \frac{8+15}{18} = \frac{23}{18} \quad \triangleright$$

$$\left[\frac{-5\sqrt{2}}{\frac{1}{\frac{7\sqrt{2}}{3\sqrt{3}}}}\right]^{-2} = \left[\frac{\frac{7\sqrt{2}}{3\sqrt{3}}}{\frac{-5\sqrt{2}}{3}}\right]^2 = \left[\frac{7\sqrt{2}}{3\sqrt{3}} \times \frac{3}{-5\sqrt{2}}\right]^2 = \left[\frac{7\sqrt{2}}{3\sqrt{3}} \times \frac{3}{-5\sqrt{2}}\right]^2 \quad \triangleright$$

$$= \left[-\frac{7\sqrt{2} \times 3}{3\sqrt{3} \times 5\sqrt{2}}\right]^2 = \left[-\frac{7}{5\sqrt{3}}\right]^2 = \frac{49}{75} \quad \triangleright$$

$$(-2\sqrt{5})^{-2} \times (-2\sqrt{5})^3 = (-2\sqrt{5})^{-2+3} = (-2\sqrt{5})^1 = -2\sqrt{5} \quad \triangleright$$

$$\left(\frac{2}{5}\right)^{-1} \times \left(-\frac{5}{2}\right)^{-4} = \left(\frac{2}{5}\right)^{-1} \times \left(-\frac{2}{5}\right)^4 = \left(\frac{2}{5}\right)^{-1} \times \left(\frac{2}{5}\right)^4 \quad \triangleright$$

$$= \left(\frac{2}{5}\right)^{-1+4} = \left(\frac{2}{5}\right)^3 = \frac{8}{125} \quad \triangleright$$

$$5^{-2} - (-\sqrt{5})^{-2} = \frac{1}{5^2} - \frac{1}{(-\sqrt{5})^2} = \frac{1}{25} - \frac{1}{5} = \frac{1}{25} - \frac{5}{25} = -\frac{4}{25} \quad \triangleright$$

$$\frac{(-2\sqrt{3})^{-2}}{\left(\frac{18}{7}\right)^{-2}} = \left[\frac{-2\sqrt{3}}{\frac{18}{7}}\right]^{-2} = \left[-2\sqrt{3} \times \frac{7}{18}\right]^{-2} = \left[\frac{-7\sqrt{3}}{9}\right]^{-2} \quad \triangleright$$

$$= \left[-\frac{9}{7\sqrt{3}}\right]^2 = \frac{81}{49 \times 3} = \frac{27}{49} \quad \triangleright$$

التعريف 4-1

احسب العبارات التالية :

$$A = [(-\sqrt{5})^{-2}]^3 \times \left[\left(\frac{1}{5\sqrt{2}}\right)^{-2} - (3\sqrt{5})^2\right] \quad \triangleright$$

$$= \left[\frac{1}{(-\sqrt{5})^2}\right]^3 \times \left[(5\sqrt{2})^2 - (3\sqrt{5})^2\right] = \left(\frac{1}{5}\right)^3 \times [50 - 45] = \frac{1}{125} \times 5 = \frac{1}{25} \quad \triangleright$$

$$B = 9 \times \left(-\frac{3}{2}\right)^{-2} - 4 \times \left(\frac{2}{3}\right)^{-3} \times 27^{-1} = 9 \times \left(-\frac{2}{3}\right)^2 - 4 \times \left(\frac{3}{2}\right)^3 \times \frac{1}{27} \quad \triangleright$$

$$= 9 \times \frac{4}{9} - 4 \times \frac{27}{8} \times \frac{1}{27} = \frac{9 \times 4}{9} - \frac{4 \times 27 \times 1}{8 \times 27} = 4 - \frac{1}{2} = \frac{8}{2} - \frac{1}{2} = \frac{7}{2} \quad \triangleright$$

$$C = \left[\frac{-\frac{5}{2}+1}{\frac{5}{4} \times 3}\right]^{-3} = \left[\frac{-\frac{5+2}{2}}{\frac{15}{4}}\right]^{-3} = \left[\frac{-\frac{3}{2}}{\frac{15}{4}}\right]^{-3} = \left[-\frac{3}{2} \times \frac{4}{15}\right]^{-3} \quad \triangleright$$

$$= \left[-\frac{3 \times 4}{2 \times 3 \times 5} \right]^{-3} = \left[-\frac{2}{5} \right]^{-3} = \left[-\frac{5}{2} \right]^3 = -\frac{125}{8}$$

$$D = \frac{-3}{4} \times \frac{125 \times 10^{-3}}{500} + \frac{0,5 \times 10^3}{250} = -\frac{1,5 \times 125 \times 10^{-3}}{2 \times 500} - \frac{4 \times 0,5 \times 10^3}{4 \times 250} >$$

$$= \frac{-15 \times 10^{-1} \times 125 \times 10^{-3}}{1000} - \frac{2 \times 10^3}{1000}$$

$$= -1875 \times 10^{-1} \times 10^{-3} \times 10^{-3} - 2 \times 10^3 \times 10^{-3}$$

$$= -1875 \times 10^{-7} - 2$$

$$= -(0,0001875 + 2) = -2,0001875$$

$$E = \frac{28^2 \times 25^2}{(3^{-4})^2 \times 49} = \frac{(4 \times 7)^2 \times 25^2}{3^{-8} \times 7^2} = 4^2 \times 7^2 \times 25^2 \times 3^8 \times 7^{-2} >$$

$$= (4 \times 25)^2 \times 7^{2+(-2)} \times 3^8 = 100^2 \times 1 \times 3^8 = 6561 \times 10^4 = 65610000$$

$$F = \frac{2^{-6} \times 35^4}{5^5 \times 7^7} \times \frac{10^3}{14^{-3}} = 2^{-6} \times (5 \times 7)^4 \times 5^{-5} \times 7^{-7} \times 10^3 \times (2 \times 7)^3 >$$

$$= 2^{-6} \times 5^4 \times 7^4 \times 5^{-5} \times 7^{-7} \times 10^3 \times 2^3 \times 7^3$$

$$= 2^{-6+3} \times 5^{4+(-5)} \times 7^{4+(-7)+3} \times 10^3 = 2^{-3} \times 5^{-1} \times 7^0 \times 10^3$$

$$= \frac{10^3}{2^3 \times 5} = \frac{10^3 \times 5^2}{2^3 \times 5 \times 5^2} = \frac{10^3 \times 25}{10^3} = 25$$

$$G = \frac{2^{10} \times 125^2 \times 20}{16 \times (-5)^8} = \frac{2^{10} \times (5^3)^2 \times 4 \times 5}{2^4 \times 5^8} = 2^{10} \times 5^6 \times 2^2 \times 5^1 \times 2^{-4} \times 5^{-8} >$$

$$= 2^{10+2+(-4)} \times 5^{6+1+(-8)} = 2^8 \times 5^{-1} = \frac{2^8}{5} = \frac{256}{5}$$

التعريف 5-11

لتكن العبارة $A = \frac{(a^{-1}b^{-2})^{-3}(ab^2)^{-2}}{(ab)^4}$ حيث a و b عدنان حقيقيان مخالفان لصفر

(1) بين أن $A = a^{-3}b^{-2}$

$$A = \frac{(a^{-1}b^{-2})^{-3}(ab^2)^{-2}}{(ab)^4} = (a^{-1})^{-3}(b^{-2})^{-3}a^{-2}(b^2)^{-2}a^{-4}b^{-4}$$

$$= a^3b^6a^{-2}b^{-4}a^{-4}b^{-4} = a^{3+(-2)+(-4)}b^{6+(-4)+(-4)} = a^{-3}b^{-2}$$

$$A = a^{-3}b^{-2} \quad \text{إذن}$$

(2) احسب A إذا علمت أن $a = \frac{1}{\sqrt{2}}$ و $b = 2\sqrt{2}$

لدينا $A = a^{-3}b^{-2}$ و $a = \frac{1}{\sqrt{2}}$ و $b = 2\sqrt{2}$

$$A = \left(\frac{1}{\sqrt{2}}\right)^{-3} (2\sqrt{2})^{-2} = (\sqrt{2})^3 \times 2^{-2} \times (\sqrt{2})^{-2} = (\sqrt{2})^{3+(-2)} \times \frac{1}{2^2}$$

التمرين 6-6

لتكن العبارة $A = \frac{8a^3(b^{-2})^3}{2a^2b^{-4}}$ حيث a و b عدنان حقيقيان مخالفان لـ صفر

(1) بين أن $A = 2^2ab^{-2}$

$$A = \frac{8a^3(b^{-2})^3}{2a^2b^{-4}} = \frac{8}{2} \times \frac{a^3}{a^2} \times \frac{b^{-6}}{b^{-4}} = 4a^{3-2}b^{-6-(-4)} = 4a^1b^{-2} = 2^2ab^{-2}$$

إذن $A = 2^2ab^{-2}$

(2) احسب A إذا علمت أن $a = \frac{3}{2}$ و $b = -7\sqrt{2}$

$$A = 2^2ab^{-2} = 4 \times \frac{3}{2} \times (-7\sqrt{2})^{-2} = 4 \times \frac{3}{2} \times \frac{1}{49 \times 2} = \frac{4 \times 3}{2 \times 49 \times 2} = \frac{3}{49}$$

إذن $A = \frac{3}{49}$

(3) لتكن العبارة $B = \frac{1}{4}a^{-1}b^2$ بين أن A و B مقلوبان

لذلك نحسب AB

$$AB = (2^2ab^{-2}) \left(\frac{1}{4}a^{-1}b^2\right) = \left(4 \times \frac{1}{4}\right) (aa^{-1})(b^{-2}b^2) = 1 \times 1 \times 1 = 1$$

إذن $AB = 1$ وبالتالي A و B مقلوبان

(4) استنتج حساب العبارة B إذا علمت أن $a = \frac{3}{2}$ و $b = -7\sqrt{2}$

بما أن A و B مقلوبان مهما يكن a و b فإن $B = \frac{1}{A}$

$$B = \frac{1}{A} = \frac{1}{\frac{3}{49}} = \frac{49}{3} \text{ إذن } b = -7\sqrt{2} \text{ و } a = \frac{3}{2}$$

إذن $B = \frac{49}{3}$

التمرين 7-7

اكتب العبارات التالية في صورة قوة للعدد 10 دليلها صحيح نسبي

$$x = \frac{\left(\frac{1}{10}\right)^{-5} \times 100^3}{(0,0001)^{-4} \times 100^{-1}} = \frac{10^5 \times (10^2)^3}{(10^{-4})^{-4} \times (10^2)^{-1}} = \frac{10^5 \times 10^6}{10^{16} \times 10^{-2}}$$

$$= 10^5 \times 10^6 \times 10^{-16} \times 10^2 = 10^{5+6+(-16)+2} = 10^{-3}$$

$$y = \frac{\frac{1}{1000} \times (0,001)^{-2}}{(0,0001)^{-3} \times \left(\frac{1}{1000}\right)^{-1}} = \frac{10^{-3} \times (10^{-3})^{-2}}{(10^{-4})^{-3} \times (10^{-3})^{-1}} = \frac{10^{-3} \times 10^6}{10^{12} \times 10^3}$$

$$= 10^{-3} \times 10^6 \times 10^{-12} \times 10^{-3} = 10^{-3+6+(-12)+(-3)} = 10^{-12}$$

$$z = \frac{(10^3)^3 \times (0,0001)^{-2}}{(0,01)^{-7} \times 10^5} = \frac{10^6 \times (10^{-4})^{-2}}{(10^{-2})^{-7} \times 10^5} = \frac{10^6 \times 10^8}{10^{14} \times 10^5}$$

$$= 10^6 \times 10^8 \times 10^{-14} \times 10^{-5} = 10^{6+8+(-14)+(-5)} = 10^{-5}$$

التعريف 8-11

اكتب في صيغة قوة عدد حقيقي دليلها عدد صحيح نسبي مخالف لـ 1

$$a = \left(\frac{\sqrt{5}}{2}\right)^{-4} \times \left(\frac{2}{\sqrt{5}}\right)^{-11} = \left(\frac{\sqrt{5}}{2}\right)^{-4} \times \left(\frac{\sqrt{5}}{2}\right)^{11} = \left(\frac{\sqrt{5}}{2}\right)^{-4+11} = \left(\frac{\sqrt{5}}{2}\right)^7$$

$$b = \left(\frac{4}{3}\right)^6 \times \left(-\frac{\sqrt{3}}{2}\right)^7 = \left[\left(\frac{2}{\sqrt{3}}\right)^2\right]^6 \times \left(-\frac{\sqrt{3}}{2}\right)^7 = \left[\left(-\frac{2}{\sqrt{3}}\right)^2\right]^6 \times \left(-\frac{\sqrt{3}}{2}\right)^7$$

$$= \left(-\frac{2}{\sqrt{3}}\right)^{12} \times \left(-\frac{\sqrt{3}}{2}\right)^7 = \left(-\frac{\sqrt{3}}{2}\right)^{-12} \times \left(-\frac{\sqrt{3}}{2}\right)^7$$

$$= \left(-\frac{\sqrt{3}}{2}\right)^{-12+7} = \left(-\frac{\sqrt{3}}{2}\right)^{-5}$$

$$c = \left(\frac{7}{13}\right)^{-4} \times \left(\sqrt{\frac{7}{13}}\right)^8 \times \frac{49}{169} = \left(\frac{7}{13}\right)^{-4} \times \left[\left(\sqrt{\frac{7}{13}}\right)^2\right]^4$$

$$= \left(\frac{7}{13}\right)^{-4} \times \left(\frac{7}{13}\right)^4 \times \left(\frac{7}{13}\right)^2 = \left(\frac{7}{13}\right)^{4+(-4)+2} = \left(\frac{7}{13}\right)^2$$

$$d = \frac{(\sqrt{14})^8}{(\sqrt{28})^5} = \left[\frac{\sqrt{14}}{\sqrt{28}}\right]^5 = \left[\frac{\sqrt{14}}{\sqrt{2 \times 14}}\right]^5 = \left[\frac{1}{\sqrt{2}}\right]^5 = \frac{1^5}{(\sqrt{2})^5} = \frac{1}{(\sqrt{2})^5} = \frac{1}{2^2 \sqrt{2}}$$

$$e = \frac{\left(\frac{\sqrt{3}}{2}\right)^5}{32} = \frac{\left(\frac{\sqrt{3}}{2}\right)^5}{2^5} = \left(\frac{\sqrt{3}}{2}\right)^5 = \left(-\frac{\sqrt{3}}{2} \times \frac{1}{2}\right)^5 = \frac{(-\sqrt{3})^5}{2^5}$$

$$f = (3\pi)^{15} \times (15\pi)^{-15} = (3\pi)^{15} \times \frac{1}{(15\pi)^{15}} = \frac{(3\pi)^{15}}{(15\pi)^{15}} = \left[\frac{3\pi}{15\pi}\right]^{15} = \left(\frac{1}{5}\right)^{15}$$

$$g = 3^{-2} \times (\sqrt{3})^{15} = \left[(\sqrt{3})^2\right]^{-2} \times (\sqrt{3})^{15} = (\sqrt{3})^{-4} \times (\sqrt{3})^{15}$$

$$= (\sqrt{3})^{-4+15} = (\sqrt{3})^{11}$$

$$h = [(\sqrt{3})^{-4}]^6 \times 6^{-12} = [((\sqrt{3})^2)^{-2}]^6 \times 6^{-12} = 3^{-12} \times 6^{-12} >$$

$$= (3 \times 6)^{-12} = 18^{-12}$$

$$i = \left[\left(\frac{\sqrt{7}}{2}\right)^5\right]^2 \times \left(\frac{2}{\sqrt{7}}\right) \times \left(\frac{\sqrt{7}}{2}\right)^{-4} = \left(\frac{\sqrt{7}}{2}\right)^{10} \times \left(\frac{\sqrt{7}}{2}\right)^{-1} \times \left(\frac{\sqrt{7}}{2}\right)^{-4} >$$

$$= \left(\frac{\sqrt{7}}{2}\right)^{10+(-1)+(-4)} = \left(\frac{\sqrt{7}}{2}\right)^5$$

$$j = \frac{5 \times 25^{-3} \times 10^2}{5^{-1} \times 2^4} = 5 \times (5^2)^{-3} \times (5 \times 2)^2 \times 5^1 \times 2^{-4} >$$

$$= 5 \times 5^{-6} \times 5^2 \times 5^1 \times 2^2 \times 2^{-4} = 5^{-2} \times 2^{-2} = (5 \times 2)^{-2} = 10^{-2}$$

$$k = \frac{0.16 \times 5^6}{\sqrt{2^{-4}} \times 10^{-6}} = \frac{16 \times 10^{-2} \times 5^6}{(\sqrt{2})^{-4} \times 10^{-6}} = 2^4 \times 10^{-2} \times 5^6 \times (\sqrt{2})^4 \times 10^6 >$$

$$= 2^4 \times ((\sqrt{2})^2)^2 \times 5^6 \times 10^6 = 2^4 \times 2^2 \times 5^6 \times 10^6 = 2^6 \times 5^6 \times 10^6$$

$$= (2 \times 5)^6 \times 10^6 = 10^6 \times 10^6 = 10^{6+6} = 10^{12}$$